Modelling the Correlation in the High-Frequency Noise of (Hetero-junction) Bipolar Transistors using Charge-Partitioning

J. C. J. Paasschens, R. J. Havens, and L. F. Tiemeijer

Philips Research Laboratories, Prof. Holstlaan 4, 5656 AA Eindhoven, The Netherlands
tel.: +31 40 2742210, e-mail: Jeroen.Paasschens@Philips.com

Abstract. We give a compact model formulation that takes into account the correlation between the shot noise of the *intrinsic* base and collector currents of a bipolar transistor, based on the physical relation between noise and charge partitioning. We show that this improves the noise modelling of especially the minimum noise figure. We also indicate why changing the doping and Ge-profiles can lead to a better noise performance.

I. Introduction

With the progress in optimisation of modern (hetero-junction) bipolar transistors the effect of the parasitic extrinsic regions reduces. Hence the intrinsic transistor behaviour becomes more dominant at high frequencies. This is also true for the noise behaviour of these transistors [1]. It is therefore important to verify and possibly improve the noise description of compact models like Spice-Gummel-Poon, Mextram, Hicum, and Vbic.

Recently a number of publications discussed the correlation between the base and collector current noise sources [2, 3]. In these publications the correlation between the external noise sources is discussed, using an effective base delay time. Most of this correlation is due to resistances and capacitances in the extrinsic regions of the transistor, and only a small, but non-negligible part comes from the intrinsic transistor. We show that using basic noise theory [4, 5] it is possible to give a noise model of this intrinsic transistor based on diffusion charges, and in particular on charge partitioning. Because of the direct relation to the charge model, the resulting noise model is easy to implement in compact models. A strong advantage is that the parameters can be extracted without measuring noise.

II. Previous methods

It is important to get a good idea about the noise performance of a device as early in the process as possible. Preferably this could be done without actually measuring the noise of a device, because noise measurements are time-consuming and often difficult to interpret. Recently a number of publications were devoted to determining the noise from just the DC currents and the Y-parameters [2, 3]. If accurate, this would be very useful because the S-parameter measurements needed are done on a regular basis anyhow. One of these methods is the simplified Spice method. In this method the transistor noise model consists of two noise sources for the shot-noise of the base and collector currents and a voltage noise source for the thermal noise of the base resistance, see *e.g.*, Ref. [3]. The two shot noise currents are sometimes taken to be correlated.

There are a few problems with trying to use the DC measurements and Y-parameters directly to estimate the

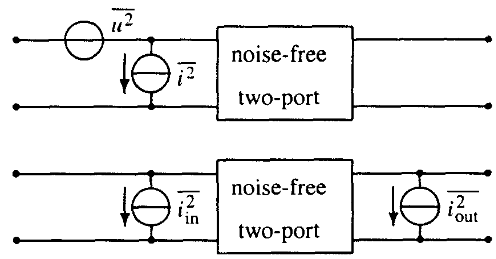

Figure 1: *Two different noise representations of a two-port. The top figure gives the chain representation, the bottom figure the admittance representation.*

noise of a device. The main problem is probably that for all but the simplest devices the noise at the terminals cannot simply be expressed directly in Y-parameters and currents of the device, without knowing some of the details of the device (like series resistances). Even though using approximate expressions can be useful, the results are not always consistent with the Y-parameters. This means that the combination of the estimated noise and the measured Y-parameters leads to unphysical results. In the calculation of the various noise representations, for instance, square roots need to be calculated. Inconsistent noise models can lead to negative values of the arguments of these square roots, as observed for instance in Ref. [3].

III. Compact model approach

As mentioned above, it is impossible to predict the noise directly for complex devices (*i.e.*, having both active and passive parts). The only viable way to make sure that the prediction is reasonably accurate and definitely consistent, is to make a detailed equivalent circuit for the transistor and make sure that all the elements are characterised and that their individual noise models are correct. The noise at the terminals is then found by calculating the transfer from each individual noise source to the terminals, taking into account all small-signal conductances and capacitances of the equivalent circuit. The external Y-parameters are calculated using the same transfer from each element to the terminals. The easiest way to do this is to use a circuit simulator. By construction this method is consistent as long as the noise sources of all elements are correctly modelled.

Since this approach is along the lines of what is normally done in compact modelling of bipolar transistors, we will concentrate on that. In a compact model there are normally three kinds of noise sources: flicker noise sources (which we will not discuss), thermal noise sources for each resistor of the equivalent circuit and shot noise sources for all diode-like elements including the main collector current. All resistors are supposed to be bulk elements, with their noise sources given by the standard thermal noise ex-

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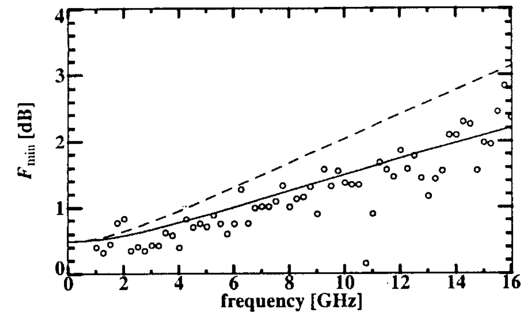

Figure 2: *Minimum noise figure F_{min} as function of frequency. Markers are from measurements at $I_C = 570\,\mu A$ and $V_{CE} = 2\,V$. The dashed line is from Mextram simulations without correlation between intrinsic base and collector currents. The solid line is with this correlation.*

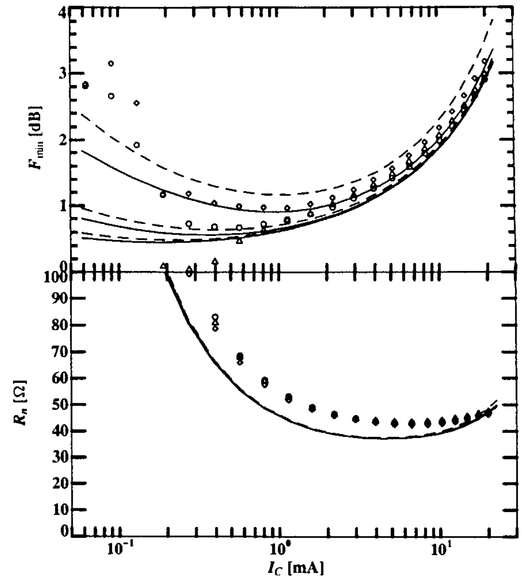

Figure 3: *Minimum noise figure F_{min} and noise resistance R_n as function of collector current at $V_{CE} = 2\,V$. Markers, dashed and solid lines are the same as in Fig. 2. Circles are for $f = 1\,GHz$, triangles are for $f = 2\,GHz$, and diamonds are for $f = 5.5\,GHz$.*

pressions. What remains is to study the noise model of the intrinsic transistor, *i.e.*, without all surrounding depletion capacitances (which do not add noise) and resistances.

To be able to study the intrinsic noise model the simulated Y-parameters must match the measured ones. This is needed for accurate calculation of the various noise representations. Moreover, the Y-parameters give an indication of the accuracy of the various (parasitic) elements in the equivalent circuit. As mentioned above, to be able to study the intrinsic noise model we must be able to accurately transfer the intrinsic noise to the terminals. This can only be done using accurate values for the elements of the equivalent circuit, that can only be obtained after parameter extraction and geometrical scaling of a set of devices with different layouts.

IV. Experimental results

The noise of a two-port can be represented — at each bias point and at each frequency — by four real numbers. There are various choices possible for the representation [6]. For designers the most relevant choice consists of the minimum noise figure F_{min}, the noise resistance R_n, and the real and imaginary part of the optimal impedance Y_{opt}. These quantities can not be related directly to effective extrinsic noise sources, in contrast to for instance the noise quantities in the admittance representation, see Fig. 1, where the noise of the two-port is given by the input current noise density $S_{i_{in}} = \overline{i_{in} i_{in}^*}/\Delta f$, the output current noise density $S_{i_{out}} = \overline{i_{out} i_{out}^*}/\Delta f$, and the correlation noise density $S_{i_{in} i_{out}} = \overline{i_{in} i_{out}^*}/\Delta f$ or the correlation coefficient $c = S_{i_{in} i_{out}}/\sqrt{S_{i_{in}} \cdot S_{i_{out}}}$. This correlation coefficient is a complex number not larger than 1 in absolute value. A third representation often used is the chain representation, in which the four noise parameters are given by S_{uu}, S_{ii} and the complex correlation admittance Y_{cor}. All these representations can be calculated from one-another using the Y-parameters of the device [6].

We used a SiGe transistor having a (drawn) emitter-size of $0.5 \times 20.3\,\mu m^2$ and a double-sided base contact from the Philips QUBiC4G process [7]. For this process a fully scaled library is available. RF noise was measured using an HP8970 noise figure meter for a number of pre-characterised source impedances. This allows us to measure a large number of points and determine the accuracy of noise measurements, see *e.g.* Fig. 2. Using open and short de-embedding structures the Y-parameters of the device, measured using an HP8510C network analyser, and noise sources are de-embedded to DUT level [8].

The measurements of the minimum noise figure as function of frequency are shown in Fig. 2. The minimum noise figure F_{min} as function of bias is plotted together with R_n and Y_{opt} in Figs. 3 and 4. For the same measurements we have also shown the noise parameters in the admittance representation (Figs. 5 and 6). The measurements become inaccurate below approximately $I_C = 0.5\,mA$, because the optimum impedance is too far from $50\,\Omega$, the impedance of our measurement setup.

In Figs. 2–6 we have also shown the results of the current Mextram model (dashed lines) [9]. In the admittance representation, Figs. 5 and 6, the simulations are close to the measurements. The current Mextram model does not include correlation between the intrinsic base and collector shot noise terms. One should note, however, that even without such an explicit correlation term the input and output current noise densities already show a large correlation. This correlation is due to the noise of the base resistance and the feedback from collector to base via the depletion capacitances. Even though the model seems to be able to accurately predict the noise parameters in the admittance representation, the minimum noise figure is predicted to be too large, more so for higher frequencies ($\gtrsim 2\,GHz$, see Fig. 2). Careful analysis shows that even a small amount of extra correlation reduces the minimum noise figure at higher frequencies. It is this correlation that we must add to the noise model of the intrinsic transistor.

V. Correlation based on Y-parameters

White noise sources in diodes and bipolar transistors are all based on microscopic diffusion noise (and possibly recombination noise), mainly in the quasi-neutral regions [4, 5].

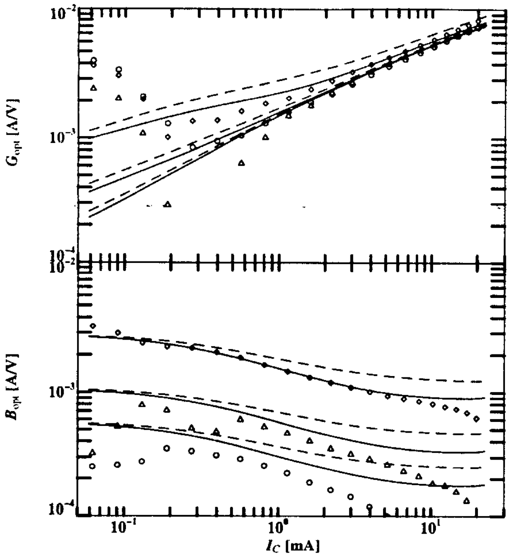

Figure 4: *Real and imaginary parts of the optimum impedance Y_{opt} as function of collector current. All markers and lines have the same meaning as in Fig. 3.*

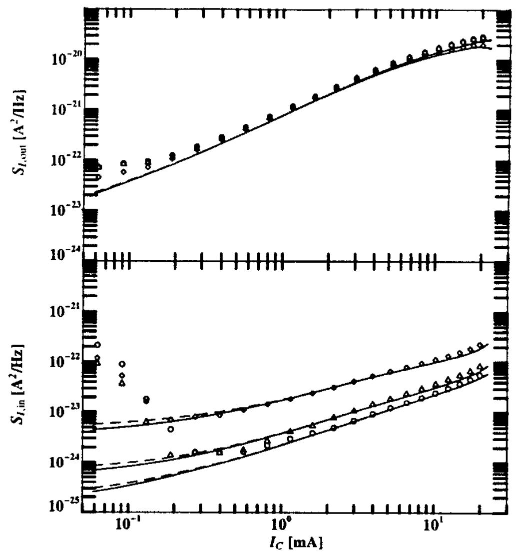

Figure 5: *Input and output noise current densities $S_{i_{in}}$ and $S_{i_{out}}$ as function of current. All markers and lines have the same meaning as in Fig. 3.*

This includes the sources like $2qI_C$, that due to their form are often called shot noise. Due to charging effects in these quasi-neutral regions the noise of *e.g.*, a diode becomes frequency dependent: $S_I = 4kT\,\Re e(Y) - 2qI$ (at low injection and without resistances), with Y the complex impedance of the diode.[1] In the low-frequency limit we can write $Y = I/V_T = qI/kT$ and hence $S_I = 2qI$, a well-known relation. Furthermore, since the depletion capacitance gives a completely imaginary contribution to the conductance, it does not give a contribution to the noise. Only higher order charging effects (second order and higher in ω) lead to an increased diode noise. Normally these non-quasi-static effects will not be modelled in a compact model: they are only important at frequencies beyond the maximum cut-off frequency.

For the *intrinsic* part of a bipolar transistor at low injection an equally general relation can be derived for the noise of the base and collector current and their correlation [12]. Neglecting Early effects, one gets [13, 14]

$$\begin{aligned} S_{i_B} &= 4kT\,\Re e\,Y_{11} - 2qI_B, \\ S_{i_C} &= 2qI_C, \\ S_{i_Bi_C} &= 2kT(Y_{21}^* - g_{m0}). \end{aligned} \quad (1)$$

Here all common-emitter Y-parameters are for the intrinsic device, and g_{m0} is the low-frequency limit of the transconductance Y_{21}.

VI. Extension of the compact model

In Eq. (1) we have given a general relation for the noise in the intrinsic transistor model based on Y-parameters. In a compact model these Y-parameters are not available directly. Only currents and charges are available. This means that only effects up to the first order in frequency are described. It is therefore of no use to describe the noise model more accurately than that. Keeping only the first order in frequency, we get the following simplification:

$$S_{i_B} = 2qI_B;\ S_{i_C} = 2qI_C;\ S_{i_Bi_C} = -2kTj\,\Im m\,Y_{21}. \quad (2)$$

The third term, $S_{i_Bi_C}$, is not included currently available compact models. This term is zero at DC and then increases with frequency. It is therefore only important at higher frequencies (but still below maximum f_T). The term $\Im m\,Y_{21}$ describes the charging current via the collector due to a change in base-emitter voltage. It does, therefore, not contain the base-collector depletion capacitance. It only contains part of the total diffusion charge Q_{tot} that is reclaimable by the collector. Hence our noise model is directly related to charge partitioning.

In charge partitioning a part of the total diffusion charge is partitioned between the emitter and the collector. Both currents can then be given by [15, 16]

$$\begin{aligned} I_E &= I_{DC} + (1-\alpha_{cp})\,dQ_{tot}/dt, \\ I_C &= I_{DC} - \alpha_{cp}\,dQ_{tot}/dt, \end{aligned} \quad (3)$$

where I_{DC} is the DC current. The net built-up of charge is $I_E - I_C = dQ_{tot}/dt$, as expected. The factor α_{cp} is the charge partitioning factor and has a value between 0 and 1. For a constant doping profile $\alpha_{cp} = 1/3$. In general this value will be larger and might even be close to 1, due to either a doping gradient or a Ge-profile [16].

In Fig. 7 show the phase of $Y_{21}^{(ext)}$ versus frequency. Note that here all extrinsic regions are taken into account. Even without charge partitioning the model is close. Taking $\alpha_{cp} = 0.5$ in the model gives a perfect fit. This small improvement is sometimes called the excess phase shift.

Based on charge partitioning, we can now define how the correlation should be modelled. At low injection we have $\Im m\,Y_{21} \simeq -\omega\alpha_{cp}Q_{tot}/V_T$, leading to

$$S_{i_Bi_C} = 2q\,j\omega\alpha_{cp}Q_{tot}. \quad (4)$$

This noise model needs one extra parameter, α_{cp}. In Mextram α_{cp} already exists, but it has the constant value 1/3

[1] Note the essential difference with the thermodynamic model [10] giving $S_I = 4kT\,\Re e\,Y + 2qI$, leading to an incorrect value of $6qI$ at low frequencies. Ref. [11] discusses why the thermodynamic method is incorrect for BJTs and HBTs.

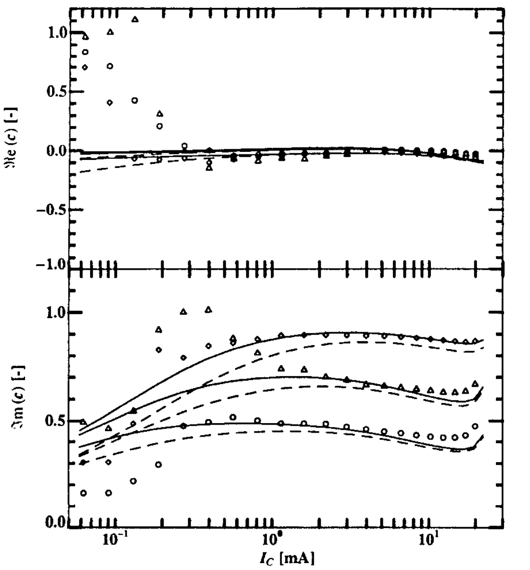

Figure 6: *Real and imaginary part of the correlation c between input and output noise current densities. All markers and lines have the same meaning as in Fig. 3.*

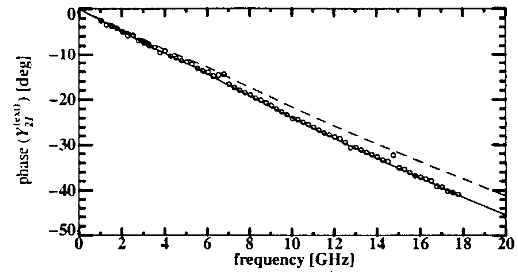

Figure 7: *Phase of the extrinsic* $Y_{21}^{(\mathrm{ext})}$ *as function of frequency. Markers are measurements. The lines are the model without (dashed) and with (solid) charge partitioning* ($\alpha_{\mathrm{cp}} = 0.5$)*.*

that should be correct in case of high injection in the base. In modern processes, however, this does not happen very often, due to the high base doping. For flexibility it is therefore useful to make α_{cp} a parameter. It can be extracted from the phase of $Y_{21}^{(\mathrm{ext})}$, but only when all phase shift due to extrinsic capacitances and resistances has been taken into account. The same parameter can then be used for the noise model. This has the large advantage that no noise measurements are needed for parameter extraction.

We have to take care that our model is consistent in all cases, especially at frequencies beyond the maximum cut-off frequency (even though the model will be inaccurate at these frequencies). For consistency we demand $|c| < 1$, or $S_{i_B} > |S_{i_B i_C}|^2/S_{i_C} = 2q(\omega\alpha_{\mathrm{cp}}Q_{\mathrm{tot}})^2/I_C$. We therefore add an extra term to S_{i_B} to make the model consistent

$$S_{i_B} = 2qI_B + 2q(\omega\alpha_{\mathrm{cp}}Q_{\mathrm{tot}})^2/I_C. \qquad (5)$$

It must be noted that this extra term does not have a physical basis. It is not a result from the $\Re e\, Y_{11}$ term, simply because we do not know that term to second order in ω in our compact model. In implementing Eq. (5) one must take care of the limit $I_C \to 0$, to prevent dividing by zero.

In Figs. 2–6 we show the results of our new noise model (solid lines). The prediction for the correlation coefficient has improved (Fig. 6) without changing the value of the input and output noise current densities (Fig. 5). This results in better fits for the optimum impedance, but especially for the minimum noise figure at higher frequencies.

VII. Discussion

We have verified the thermal and shot noise model of the compact model Mextram. We found that the minimum noise figure could be modelled accurately when the correlation between the base and collector shot noise terms of the intrinsic transistor is accounted for correctly. We have given a compact model formulation for this correlation, that can be used in any model. To do this we did not use an empirical delay time. Instead we used general noise expressions to make a link to charge partitioning. Using the charge partitioning it is then possible to elegantly describe the correlation. Due to the intimate link, the parameter needed for the noise model is the same as needed for charge partitioning. It can therefore be extracted from the excess phase shift, which can be found from S-parameters that are measured regularly.

Our analysis also gives a reason why changing doping and Ge-profiles can be used to optimise the noise behaviour. We showed that a lower noise figure results from an increased correlation, which is directly related to the charge partitioning factor α_{cp}. Increasing this factor will reduce the noise figure. Hence, the profile has to be designed such that most of the diffusion charge is reclaimable by the collector, not the emitter. This is why the profiles given in for instance Ref. [17] can give better noise figures, even when the DC current gain is fixed.

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