Improved Extraction of Base and Emitter Resistance from Small Signal High Frequency Admittance Measurements

W.J. Kloosterman, J.C.J. Paasschens and D.B.M. Klaassen

Philips Research Laboratories, Prof. Holstlaan 4, 5656 AA Eindhoven, The Netherlands
tel.: +31 40 2744093, email: kloosteb@natlab.research.philips.com

Abstract

The base- and emitter resistance are important parameters of bipolar transistors and can be extracted from high frequency small signal admittance (Y) parameters. The circle impedance method and the two port method are examined and improvements are presented.

1. Introduction

The base resistance is an important parameter of bipolar transistors and it strongly influences device performance. The base resistance can be extracted from both DC [1] and AC [2] measurements. The Ning & Tang method [1] gives good results when the emitter resistance is small ($R_e \cdot \beta < R_b$) and the degradation of the current gain β is due to high injection in the base. With decreasing transistor dimensions the emitter resistance increases whereas the base resistance may decrease. The emitter resistance adds to the base resistance in AC and noise measurements. This makes that the extraction of the base resistance from AC and noise measurements becomes more favorable for advanced bipolar transistors and is in many cases the only way to extract accurate values. High frequency admittance measurements are performed already to determine the cut off frequency fT of the bipolar transistor. From the same measurements we can extract the base and emitter resistance. We will examine two commonly used methods being the circle impedance method and the two port method. The influence of the intrinsic, the extrinsic base-collector capacitance and emitter resistance on the extracted base resistance in both methods is shown. By using the DC collector and base current and just one point in the frequency domain reliable base and emitter resistances can be extracted. Experimental results of an NPN transistor in the QUBiC3 process [4] are shown and compared with the Ning & Tang method.

2. Common emitter admittance matrix

In this section the admittance matrix is derived for a relative simple equivalent transistor circuit (see figure 1). The intrinsic transistor is represented by the base-emitter conductance g_0, the base-emitter capacitance by C_b, the base-collector capacitance underneath the emitter by C_{in} and the transconductance by g_m. The total base resistance is given by R_b, the emitter resistance by R_e and the extrinsic base-collector capacitance by C_{ex}. The common emitter admittance matrix is defined as;

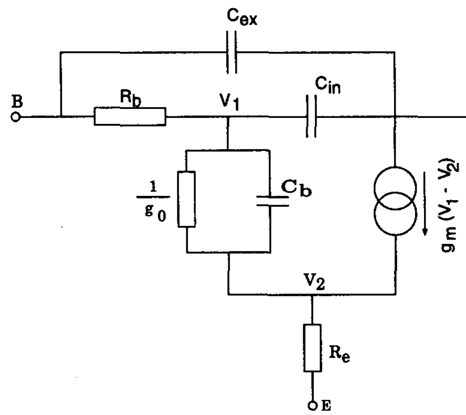

Figure 1: *Small signal equivalent transistor circuit for deriving the common emitter admittance matrix.*

$$\begin{pmatrix} i_b \\ i_c \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} \begin{pmatrix} v_b \\ v_c \end{pmatrix}$$

It is convenient to define first the admittance matrix without the the base resistance R_b and the extrinsic capacitance C_{ex}. This admittance matrix reads;

$$\begin{aligned} Y_{11}^{Re} &= \frac{g_0 + j \cdot \omega \cdot C_b}{1 + F_{Re}} + j \cdot \omega \cdot C_{in} \\ Y_{21}^{Re} &= \frac{g_m}{1 + F_{Re}} - j \cdot \omega \cdot C_{in} \\ Y_{12}^{Re} &= -j \cdot \omega \cdot C_{in} \\ Y_{22}^{Re} &= +j \cdot \omega \cdot C_{in} \\ F_{Re} &= (g_m + g_0 + j \cdot \omega \cdot C_b) \cdot R_e \end{aligned}$$

In the next step we add the base resistance R_b and the extrinsic capacitance C_{ex};

$$Y_{11} = \frac{Y_{11}^{Re}}{1 + Y_{11}^{Re} \cdot R_b} + j \cdot \omega \cdot C_{ex} \quad (1)$$

$$Y_{21} = \frac{Y_{21}^{Re}}{1 + Y_{11}^{Re} \cdot R_b} - j \cdot \omega \cdot C_{ex} \quad (2)$$

$$Y_{12} = \frac{Y_{12}^{Re}}{1 + Y_{11}^{Re} \cdot R_b} - j \cdot \omega \cdot C_{ex} \quad (3)$$

$$Y_{22} = Y_{22}^{Re} \cdot \left(1 + \frac{Y_{21}^{Re} \cdot R_b}{1 + Y_{11}^{Re} \cdot R_b}\right) + j \cdot \omega \cdot C_{ex} \quad (4)$$

0-7803-5712-4/99/$10.00 ©1999 IEEE

The above expressions are used to evaluate the base and emitter resistance extraction methods.

3. The base resistance extraction

There are two standard methods to extract the base resistance from the emitter admittance matrix. The first one is the circle impedance method and the second one is the two port method.

3.1 Circle impedance method

In the equivalent circuit (figure 1) of the standard circle impedance method the emitter resistance R_e and the external collector capacitance C_{ex} are zero and the input impedance $H_{11} = 1/Y_{11}$ then becomes;

$$H_{11} = R_b + \frac{1}{g_0 + j \cdot \omega \cdot (C_b + C_{in})} \tag{5}$$

The plot of H_{11} in a complex plane is a semi-circle (fig. 2: solid line). The extrapolated intersection with the x-axis

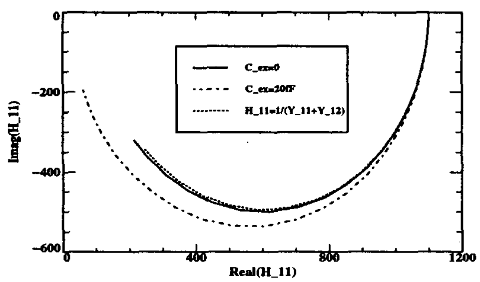

Figure 2: *Plots of H_{11} in the complex plane. The parameters of the equivalent circuit are: $R_b = 100$, $g_0 = 0.001$, $C_b = 20fF$, $C_{in} = 2fF$, $R_e = 0$ and C_{ex} is varied.*

gives the base-resistance $R_b = 100\Omega$. When we add an external base-collector capacitance C_{ex} to the equivalent circuit the extrapolated intersection is not the base resistance (see fig. 2). Due to the extrinsic base-collector capacitance we will underestimate the base resistance (in this example R_b becomes 24Ω). We can improve the extraction of the base resistance [2] by adding the Y_{12} component to Y_{11} and apply then the circle impedance method:

$$\begin{aligned} Y_{11}^i &= Y_{11} + Y_{12} \\ H_{11}^i &= \frac{1}{Y_{11}^i} = \frac{1 + R_b \cdot (g_0 + j \cdot \omega\,(C_b + C_{in}))}{g_0 + j \cdot \omega \cdot C_b} \end{aligned} \tag{6}$$

For $\omega \Rightarrow \infty$ we can compute R_b;

$$R_b = \frac{C_b}{C_b + C_{in}} \cdot H_{11}^i(\omega \Rightarrow \infty) \tag{7}$$

In most cases $C_b \gg C_{in}$ and the intersection with the x-axis gives a good estimate of the actual base resistance. The term $C_b/(C_b + C_{in})$ can be estimated from the Y parameters as follows;

$$\frac{\omega \cdot C_b}{g_m} = \mathrm{Im}\left(\frac{\mathrm{Y}_{11} + \mathrm{Y}_{12}}{\mathrm{Y}_{21} - \mathrm{Y}_{12}}\right) \tag{8}$$

$$\frac{\omega \cdot C_{in}}{g_m} = \frac{\mathrm{Re}(\mathrm{Y}_{12})}{\mathrm{Im}\,(\mathrm{Y}_{21} - \mathrm{Y}_{12})} \tag{9}$$

When two points (x_0, y_0 and x_1, y_1) of the circle are known we can directly calculate R_b [3];

$$\begin{aligned} x &= \mathrm{Re}(\mathrm{H}_{11}), \quad \mathrm{y} = \mathrm{Re}(\mathrm{H}_{11})^2 + \mathrm{Im}(\mathrm{H}_{11})^2 \\ m &= \frac{y_1 - y_0}{x_1 - x_0}, \quad b = y_1 - m \cdot x_1 \\ R_b &= \frac{m}{2} - \sqrt{b + m^2/4} \end{aligned} \tag{10}$$

The two points may be the DC point (frequency=0) and just one point in the frequency domain.

3.2 Two port method

In the equivalent circuit of the standard two port method the emitter resistance R_e and the internal collector capacitance C_{in} are zero. From the Y parameters as defined in section 2 the base resistance then becomes;

$$R_b = \frac{1}{g_m} \cdot \frac{g_m + Y_{12} - Y_{21}}{Y_{11} + Y_{12}} \tag{11}$$

The influence of the internal b-c capacitance C_{in} follows from the Y parameters as defined in section 2;

$$R_b = \frac{1}{g_m} \cdot \frac{g_m + Y_{12} - Y_{21}}{Y_{11} + Y_{12}} \cdot \frac{g_0 + j \cdot \omega \cdot C_b}{g_0 + j \cdot \omega \cdot (C_b + C_{in})} \tag{12}$$

The base resistance now becomes frequency dependent. At sufficient high frequency R_b is;

$$R_b \approx \frac{1}{g_m} \cdot \frac{g_m + Y_{12} - Y_{21}}{Y_{11} + Y_{12}} \cdot \frac{C_b}{C_b + C_{in}} \tag{13}$$

In the two port method we need besides the Y parameters also the value of g_m (see section 4). Due to the internal base-collector capacitance C_{in} the standard two port method overestimates the base resistance. The correction term is frequency dependent and at high frequency it is the same as derived for the circle impedance method. The base resistance R_b will be complex when relation (11) is applied to measured Y parameters. The extracted base resistance then should read $R_b^m = 1/\mathrm{Re}(1/\mathrm{R_b})$.

4. The emitter resistance

In the previous section the emitter resistance is neglected. The emitter resistance can be calculated from the difference between the intrinsic conductance g_m of the collector current and the measured external conductance g_m^m. The reduced small signal conductance g_m^m due to the base and emitter resistance is given by (see section 2);

$$g_m^m = \frac{g_m}{1 + g_m \cdot R_e + g_0 \cdot (R_e + R_b)} \tag{14}$$

and the emitter resistance becomes;

$$R_e = \frac{g_m - g_m^m \cdot (1 + g_0 \cdot R_b)}{g_m^m \cdot (g_m + g_0)} \tag{15}$$

The conductance g_m^m can be obtained directly from the low frequency asymptote of Y_{21} or calculated from the

Gummel plot;

$$g_m^m = \frac{I_c(n)\cdot\ln\left(\frac{I_c(n)}{I_c(n-1)}\right)}{V_{be}(n)-V_{be}(n-1)}$$

$$g_0^m = \frac{I_b(n)\cdot\ln\left(\frac{I_b(n)}{I_b(n-1)}\right)}{V_{be}(n)-V_{be}(n-1)}$$

where n and $n-1$ refer to two points of the Gummel plot. The intrinsic conductances g_0 and g_m may be calculated from the DC collector and base current. We assume that the collector current is non-ideal (slope m) and the base current is ideal;

$$I_c(n) = I_s\cdot\exp\left(\frac{V_{be}^i(n)}{m\cdot V_t}\right)$$

$$I_b(n) = \frac{I_s}{B_f}\cdot\exp\left(\frac{V_{be}^i(n)}{V_t}\right)$$

$$g_m = \frac{\partial I_c}{\partial V_{be}^i} = \frac{I_c(n)}{m\cdot V_t},\quad g_0 = \frac{\partial I_b}{\partial V_{be}^i} = \frac{I_b(n)}{V_t}$$

$$m = \frac{\beta(DC)}{\beta(AC)} = \frac{I_c}{I_b}\cdot\frac{g_0}{g_m} = \frac{I_c(n)\cdot g_0^m}{I_b(n)\cdot g_m^m} > 1$$

in which V_t is the thermal voltage, I_s the collector saturation current, B_f the forward current gain and V_{be}^i the internal base emitter junction voltage. The non ideality factor m is calculated from the difference in DC and AC current gain. In following sections we will apply the standard extraction method and then we correct for the presence of the emitter resistance. An alternative way is to de-embed the admittance matrix for the constant part of the series resistance. This can be done by converting the Y parameter to Z parameters and subtract then the emitter resistance , the constant part (or fraction) of the base resistance R_{bc} and also the constant part of the collector resistance. Then we transform the corrected Z parameters again to Y parameters.

4.1 Influence of R_e in the circle impedance method

When we include the emitter resistance in the small signal equivalent circuit the input impedance H_{11} for $\omega \Rightarrow \infty$ (R_{be}) calculated from (1) with $C_{ex} = 0$ becomes;

$$R_{be} = R_b + \left(\frac{C_b}{C_b + C_{in}\cdot(1+(g_m+g_0)\cdot R_e)}\right)^2\cdot R_e \quad (16)$$

In most cases C_b is much larger then C_{in} and the emitter resistance adds to the base resistance. With an external base-collector capacitance C_{ex} we have to use the improved circle impedance method with $H_{11}^i = 1/(Y_{11}+Y_{12})$. Also in this case we can derive the influence of the emitter resistance;

$$R_{be} = R_b + R_e + \frac{C_{in}}{C_b}\cdot\{R_b + (g_m+g_0)\cdot R_e\cdot R_b\} \quad (17)$$

We see that in the standard circle impedance method and the improved circle impedance method the influence of the emitter resistance is slightly different. In both methods we can correct the extrapolated intersection for the presence of the emitter resistance. The value of C_{in}/C_b is given by (8, 9).

4.2 Influence of R_e in the two port method

When an emitter resistance is present in the equivalent circuit the extracted resistance value R_{be} of the two port method becomes;

$$R_{be} = R_b\cdot\frac{g_0 + j\cdot\omega\cdot(C_b+C_{in})}{g_0+j\cdot\omega\cdot C_b} + R_e\cdot\frac{g_m+g_0+j\cdot\omega\cdot C_b}{g_0+j\cdot\omega\cdot C_b} \quad (18)$$

At low frequencies and up to medium collector current levels the emitter resistance is multiplied by the small signal gain $(g_m+g_0)/g_0$ of the transistor. This makes that an accurate determination of the emitter resistance is very important in the two port method. When the emitter resistance is not constant it affects the extracted base resistance more as in the circle impedance method. At sufficient high frequencies and collector current ($\omega\cdot C_b \gg g_m$) the emitter resistance adds to the base resistance.

5. Experimental results

The transistor under test is a double-poly vertical NPN transistor from the QUBiC3 process [4] with an effective emitter size of 0.3x19.8 μm^2 and with a double base contact. The collector current I_c and base current I_b are measured (Gummel plot) in the range from V_{be} is 0.8 to 1.1 Volt at constant $V_{bc} = -1$ Volt. The small signal admittance matrixes (Y parameters) are measured for the same bias points at a frequency of 3.51 Ghz. For one bias point ($V_{be} = 0.95V$,$V_{bc} = 0V$) the Y parameters are measured in a frequency sweep from 1 to 40 GHz. The DC gain and cut-off frequency (fT) are shown in figure 3. First the Ning & Tang method is applied to the

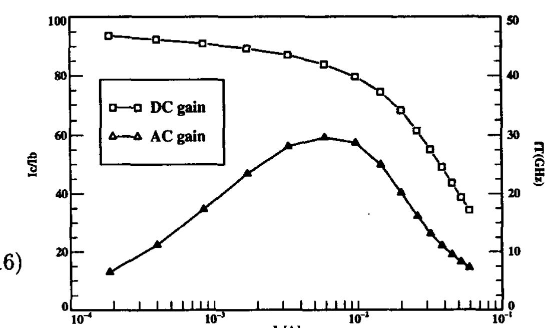

Figure 3: *measured DC current gain and fT obtained from the Gummel plot and Y parameters ($V_{bc} = -1$ Volt)*

Gummel plot and the results are shown in figure 4. The constant part of the base resistance R_{bc} and the emitter resistance with the bias dependent part $R_{bv}/Bf = 0.14$ are evaluated using two sequential points of the Ning & Tang curve. The calculated emitter resistance is almost constant whereas the base resistance (given by the slope) decreases strongly with I_b/I_c. The bad behavior of R_{bc} is due to the fact that in the Gummel plot the effect of

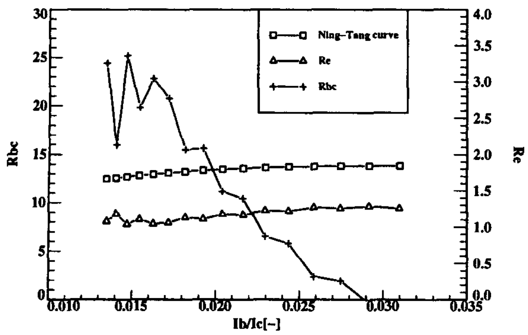

Figure 4: *Extraction of the constant part of Rb (Rb_c) and Re from the Gummel plot using the Ning & Tang method.*

emitter resistance becomes multiplied with the DC gain and mainly dominates the voltage drop between internal and external base-emitter junction. Next (fig: 5) the extraction of the base resistance from Y parameters in a frequency sweep is analysed. The emitter resistance according to (15) is calculated using g_0^m and g_m^m from Y_{11} and Y_{12} ($I_c = 12.3mA$, $\beta(DC) = 73.2$, $\beta(AC) = 60.1$, $R_e = 2.07\Omega$). In the standard two port method R_e is subtracted from the calculated R_{be} (18) to obtain R_b. The correction for R_e (improved two port method) according to (18) gives an almost constant R_b. The circle impedance method is applied using only two points being the DC point ($x_0 = \beta(AC)/g_m^m$, $y_0 = x_0^2$) and one frequency point ($x_1 = \mathrm{Re}(\mathrm{H}_{11})$, $y_1 = \mathrm{Re}(\mathrm{H}_{11})^2 + \mathrm{Im}(\mathrm{H}_{11})^2$). In the standard circle impedance method $H_{11} = 1/Y_{11}$ and R_e is subtracted from (10). In the improved circle impedance method $H_{11} = 1/(Y_{11} + Y_{12})$ and the correction for R_e is according to (17). For this bias point the term C_{in}/C_b is very small. We see that both the standard and improved circle impedance method gives an almost constant R_b. The emitter resistance simply adds to the base resistance in contrast with the two port method. In figure 6 the base resistance is calculated versus bias using at each bias point the small signal admittance matrix (at one frequency) and the Gummel plot. The emitter resistance is calculated according to section 4. It slightly decreases at high collector currents. The difference between the extracted base resistance of the four methods decreases with increasing bias. The effect of emitter resistance correction is large in the two port method. At low bias the difference between de standard and improved circle impedance method is mainly due to the modified definition of $H_{11} = 1/(Y_{11} + Y_{12})$. De-embedding first the same emitter resistance from the small signal data and applying then the various methods with $R_e = 0$ give simular results as using the derived terms for the correction of R_e.

6. Summary

In this paper the extraction of base resistance and emitter resistance from small signal admittance matrixes is examinated and improvements are given. For advanced bipolar processes the tendency in the circle method is to underestimate slightly the base resistance whereas the two port method greatly overestimate it. In the circle impedance method the emitter resistance simply adds to the base resistance. In the two port method R_e becomes multiplied with the AC current gain and therefore an accurate estimation of R_e and correction for R_e is very important. It is shown that the circle impedance method can be applied to Y parameters measured at only one frequency together with the DC Gummel plot. This greatly reduces the effort to take measurements. For advanced bipolar processes the circle impedance method is the best procedure to extract accurate base resistance parameters.

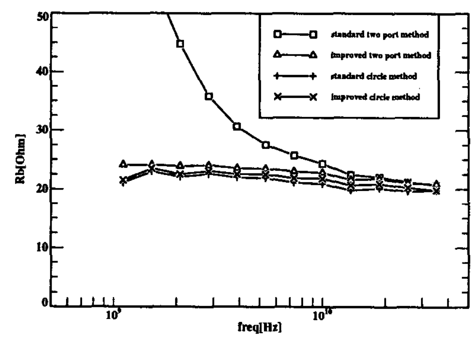

Figure 5: *Extraction of the base resistance for each frequency point from small signal Y parameters*

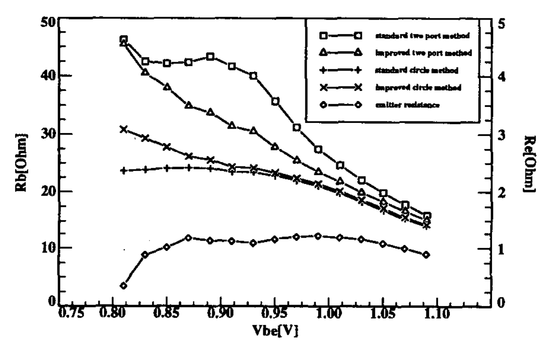

Figure 6: *Extraction of the base resistance from small signal Y parameters versus bias using just one frequency point and the Gummel plot.*

References

[1] Tak H. Ning and Denny D. Tang Method for Determining the Emitter and Base Series Resistance of Bipolar Transistors IEEE Transactions on Electron Devices Vol ED-31 No. 4 pp 409-412

[2] Takado Nakadai, Kenji Hashimoto "Measuring the Base resistance of Bipolar Transistors." BCTM 1991 paper 8.4 page 200

[3] R. Kuhn and A. Cuthbertson Esprit 2048/JessiT28 Tibia II Workpackage 3 "Process Support" Minutes of the 2nd internal workshop 14th March 1996 Londen

[4] Armand Pruijmboom, et al. "QUBiC3: a 0.5 μm BiCMOS Production Technology, with fT=30GHz, fmax=60Ghz and High-Quality Passive Components for Wireless Telecommunication Application. BCTM 1998 paper 7.1 page 120.